Triangle Theorem Proofs and Application Practice

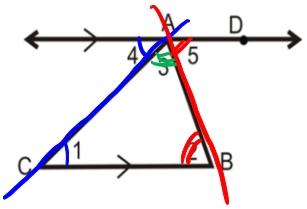
Do you remember the triangle sum theorem? What did it say?

Proof of Triangle Sum Theorem

(Interior Angle Sum Theorem)

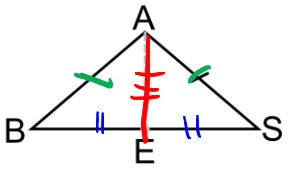
Given: ΔABC and $\overleftrightarrow{AD} \| \overleftrightarrow{CB}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statement	Reason
ΔABC	Given
$\overline{AD} \| \overline{CB}$	Given
$m\angle 4 + m\angle 5 + m\angle 3 = 180$	Def. of straight angle
$\angle 1 \cong \angle 4$	Alt. Int. $\angle$'s Thm.
$\angle 2 \cong \angle 5$	Alt. Int. $\angle$'s Thm.
$m\angle 1 = m\angle 4$	Def. of $\cong$
$m\angle 2 = m\angle 5$	Def. of $\cong$
$m\angle 1 + m\angle 2 + m\angle 3 = 180$	Substitution

Prove the Base Angles Theorem:



Given: ΔBAS is isosceles
E is the midpoint of BS

Prove: $\angle ABE \cong \angle ASE$

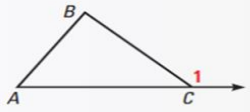
Statement	Reason
ΔBAS is isosceles	Given
E is the midpoint of BS	Given
$\overline{BA} \cong \overline{SA}$	Def. of Isosceles
$\overline{BE} \cong \overline{SE}$	Def. of midpoint
$\overline{AE} \cong \overline{AE}$	Reflexive Prop.
$\Delta ABE \cong \Delta ASE$	SSS
$\angle ABE \cong \angle ASE$	CPCTC

THEOREM

THEOREM 4.2 ***Exterior Angle Theorem***

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$m\angle 1 = m\angle A + m\angle B$



Ex 14: Given: $\angle 1$ is an exterior angle of ΔABC.

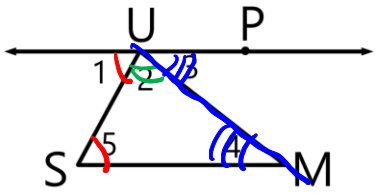
Prove: $m\angle 1 = m\angle A + m\angle B$

Statements	Reasons
1. $\angle 1$ is an exterior angle of ΔABC.	1. Given
2. $\angle ACB$ and $\angle 1$ are a linear pair	2. Definition of Exterior Angle
3. $m\angle ACB + m\angle 1 = 180°$	3. Linear Pairs conjecture
4. $m\angle A + m\angle B + m\angle ACB = 180°$	4. Triangle Sum Theorem
5. $m\angle ACB + m\angle 1 = m\angle A + m\angle B + m\angle ACB$ (subtract $m\angle ACB$)	5. Substitution
6. $m\angle 1 = m\angle A + m\angle B$	6. Inverse Prop. of addition

Triangle Theorem Proofs and Application Practice

Given: ΔSUM and $\overline{UP} \| \overline{SM}$

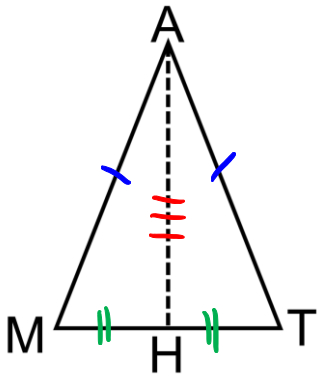
Prove: $m\angle 5 + m\angle 4 + m\angle 2 = 180^o$



Statement	Reason
ΔSUM	Given
$\overline{UP} \| \overline{SM}$	Given
$\angle 1 \cong \angle 5$ and $\angle 3 \cong \angle 4$	Alt. Int. $\angle$'s Thm.
$m\angle 1 = m\angle 5$ and $m\angle 3 \cong m\angle 4$	Def. of $\cong$
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$	Def. of straight $\angle$
$m\angle 5 + m\angle 2 + m\angle 4 = 180°$	Substitution

Given: ΔMAT *is isosceles and* H *is the midpoint of* $\overline{MT}$

Prove: $\angle AMH \cong \angle ATM$



Statement	Reason
ΔMAT is isosceles	Given
H is midpoint of $\overline{MT}$	Given
$\overline{MA} \cong \overline{TA}$	Def. of Isosceles
$\overline{MH} \cong \overline{TH}$	Def. of midpoint
$\overline{HA} \cong \overline{HA}$	Reflexive property
$\Delta MAH \cong \Delta TAH$	SSS
$\angle AMH \cong \angle ATM$	CPCTC