

Warm Up

List 5 things you think you know about triangles.

Triangle Theorem Notes

Standards for this week:

CO.10 Prove theorems about and classify triangles. Theorems include:

- measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem;
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.




Prove theorems about triangles. Theorems include:

o one side of a triangle divides the other two





Triangle Theorem Notes

NAMES OF TRIANGLES

Classification by Sides

<p>EQUILATERAL TRIANGLE</p>  <p>3 congruent sides</p>	<p>ISOSCELES TRIANGLE</p>  <p>At least 2 congruent sides</p>	<p>SCALENE TRIANGLE</p>  <p>No congruent sides</p>
---	--	--

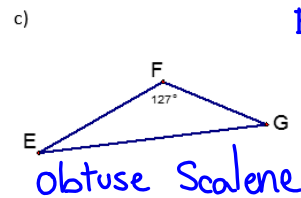
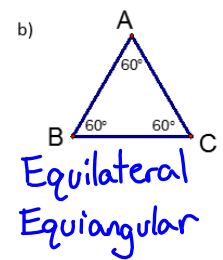
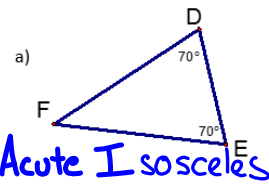
Classification by Angles

<p>ACUTE TRIANGLE</p>  <p>3 acute angles</p>	<p>EQUIANGULAR TRIANGLE</p>  <p>3 congruent angles</p>	<p>RIGHT TRIANGLE</p>  <p>1 right angle</p>	<p>OBTUSE TRIANGLE</p>  <p>1 obtuse angle</p>
--	--	---	---

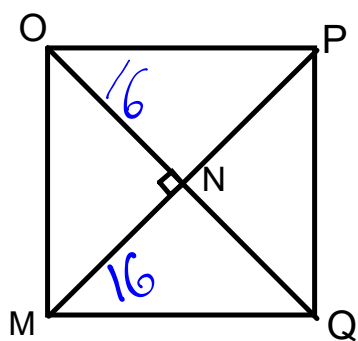
Note: An equiangular triangle is also acute.

** Equilateral triangles are also isosceles*

Ex 1: Classify the triangle by its sides and angles.



$$MN=16, NO=16$$



a. Explain why $\triangle MNO$ is an isosceles right triangle.

It is isosceles because it has 2 \cong sides. It is right because it has a 90° \angle .

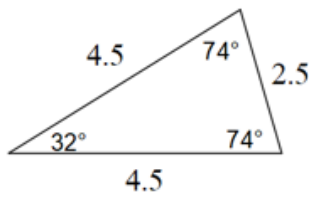
b. Identify the hypotenuse and legs of $\triangle MNO$

Hyp: \overline{MO} Leg: \overline{ON} and \overline{MN}

Triangle Theorem Notes

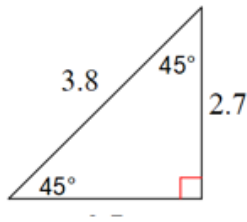
Practice classifying: 1,3,5,7

1)



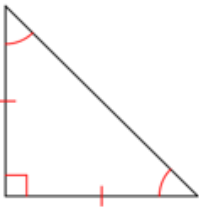
Acute Isosceles

3)



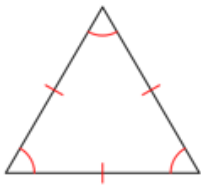
Right Isosceles

5)



Right Isosceles

7)



Equiangular
Equilateral

Standards

CO.10 Prove theorems about and classify triangles. Theorems include:

- Prove measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem;
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

SRT.4 Prove theorems about triangles. Theorems include:

- a line parallel to one side of a triangle divides the other two proportionally

Triangle Theorem Notes

Triangle Sum Theorem

Do you remember the triangle sum theorem? What did it say?

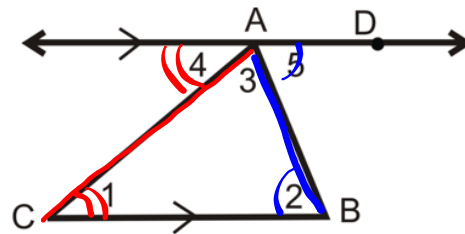
The sum of the interior angles is 180°

Proof of Triangle Sum Theorem

(Interior Angle Sum Theorem)

Given: $\triangle ABC$ and $\overline{AD} \parallel \overline{CB}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$



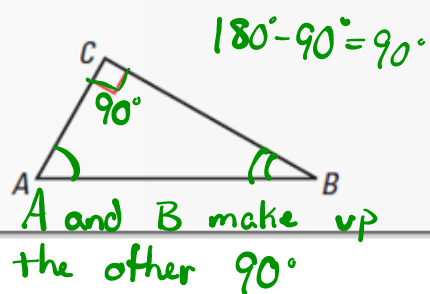
Statement	Reason
$\triangle ABC$	Given
$\overline{AD} \parallel \overline{CB}$	Given
$m\angle 4 + m\angle 5 + m\angle 3 = 180^\circ$	Def. of straight \angle
$\angle 1 \cong \angle 4$	Alt. Int. \angle 's Thm.
$\angle 2 \cong \angle 5$	Alt. Int. \angle 's Thm.
$m\angle 1 = m\angle 4$	Def. of \cong
$m\angle 2 = m\angle 5$	Def. of \cong
$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$	Substitution

COROLLARY

COROLLARY TO THE TRIANGLE SUM THEOREM

The acute angles of a right triangle are complementary.

$$m\angle A + m\angle B = 90^\circ$$



Triangle Theorem Notes

Ex 3: $\triangle ABC$ is a right triangle. $\angle A$ and $\angle B$ are acute angles. Determine the value of the acute angle in the following examples.

a) $m\angle A = 27^\circ, \angle B = \underline{63^\circ}$



$$90^\circ - 27^\circ = 63^\circ$$

c) $m\angle A = 45^\circ, \angle B = \underline{45^\circ}$

$$90^\circ - 45^\circ = 45^\circ$$

b) $m\angle A = 15^\circ, \angle B = \underline{75^\circ}$

$$90^\circ - 15^\circ = 75^\circ$$

d) $m\angle A = 74^\circ, \angle B = \underline{16^\circ}$

$$90^\circ - 74^\circ = 16^\circ$$

Priority Standards

Make 4 groups of angles that form a triangle.

Obtuse
scalene

60

95

25°

30°

60°

90°

Right
Scalene

Acute
scalene

64°

62°

54°

90°

45°

45°

Right

Isosceles

Classify the triangles you made.

Standards

CO.10 Prove theorems about and classify triangles. Theorems include:

- measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem;
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

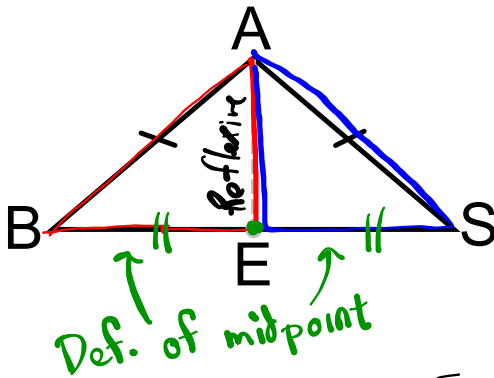
SRT.4 Prove theorems about triangles. Theorems include:

- a line parallel to one side of a triangle divides the other two proportionally

Triangle Theorem Notes

Prove the Base Angles Theorem:

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.



Given: $\overline{AB} \cong \overline{AS}$

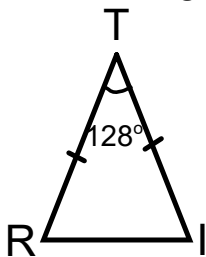
E is the midpoint of BS

Prove: $\angle ABE \cong \angle ASE$

statement	Reason
$\overline{AB} \cong \overline{AS}$	Given
E is the midpoint of \overline{BS}	Given
$\overline{AE} \cong \overline{AE}$	Reflexive Prop.
$\overline{BE} \cong \overline{SE}$	Def. of Midpoint.
$\triangle ABE \cong \triangle ASE$	SSS
$\angle ABE \cong \angle ASE$	CPCTC

Triangle Theorem Notes

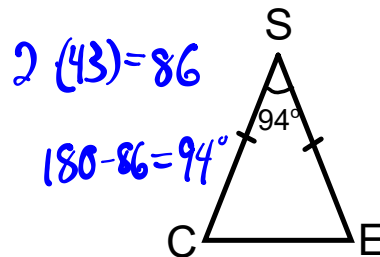
Find an Isosceles triangle whose base angles are 26° .



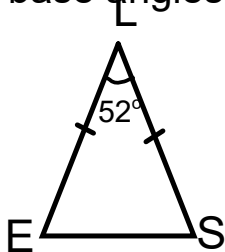
$$\begin{array}{r} 180 \\ - 128 \\ \hline 52 \end{array}$$

$$\frac{52}{2} = 26$$

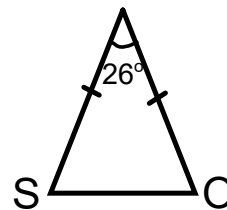
Find an Isosceles triangle whose base angles are 43° .



Find an Isosceles triangle whose base angles are 64° .



Find an Isosceles triangle whose base angles are 77° .



Standards

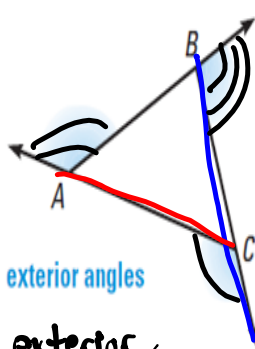
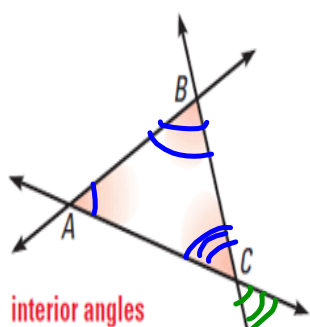
CO.10 Prove theorems about and classify triangles. Theorems include:

- measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem;
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

SRT.4 Prove theorems about triangles. Theorems include:

- a line parallel to one side of a triangle divides the other two proportionally

Triangle Theorem Notes



This is not an exterior \angle

This is an exterior \angle

Exterior Angle- the angle between any side of a shape, and a line extended from the next side.

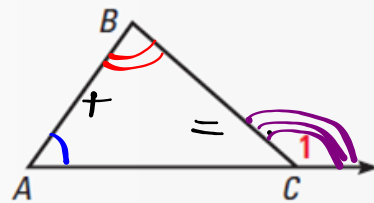
Triangle Theorem Notes

THEOREM

THEOREM 4.2 *Exterior Angle Theorem*

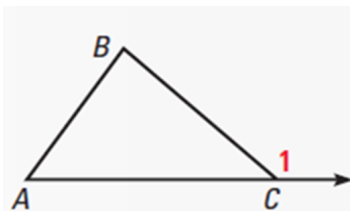
The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$$m\angle 1 = m\angle A + m\angle B$$



Ex 14: Given: $\angle 1$ is an exterior angle of $\triangle ABC$.

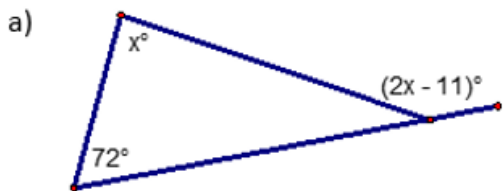
Prove: $m\angle 1 = m\angle A + m\angle B$



Statements	Reasons
1. $\angle 1$ is an exterior angle of $\triangle ABC$.	1. Given
2. $\angle ACB$ and $\angle 1$ are a linear pair	2. Definition of Exterior Angle
3. $m\angle ACB + m\angle 1 = 180^\circ$	3. Linear Pairs Conjecture
4. $m\angle A + m\angle B + m\angle ACB = 180^\circ$	4. Triangle Sum Theorem
5. $m\angle ACB + m\angle 1 = m\angle A + m\angle B + m\angle ACB$	5. Substitution
6. $m\angle 1 = m\angle A + m\angle B$	6. Inverse Prop. of addition

Triangle Theorem Notes

Ex 13: Find the value of x . then find the measure of the exterior angle.

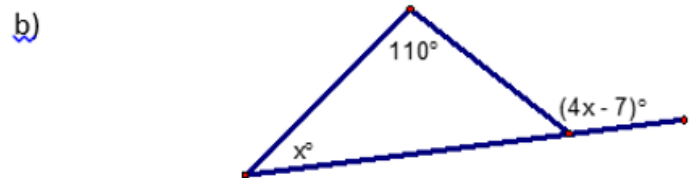


$$x + 72 = 2x - 11$$

$$72 = x - 11$$

$$x = 83$$

$$\text{Exterior } \angle = 2(83) - 11 = 155^\circ$$



$$x + 110 = 4x - 7$$

$$117 = 3x$$

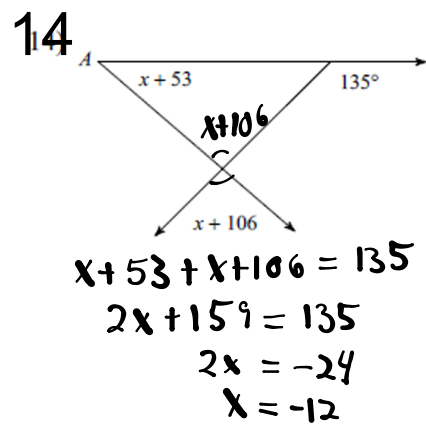
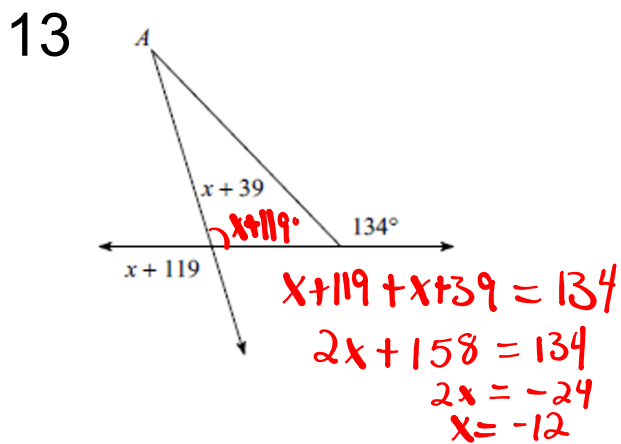
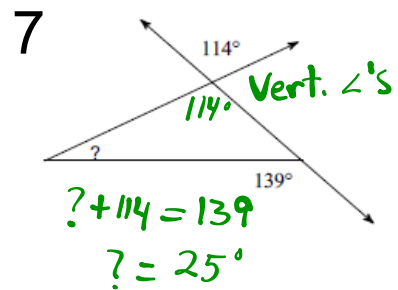
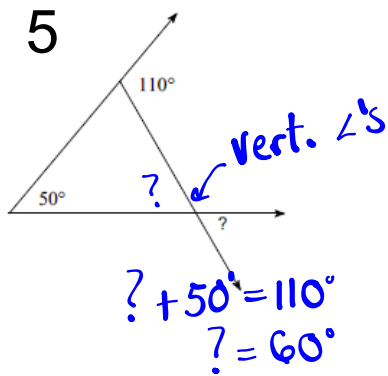
$$x = 39$$

Ext. \angle

$$\underline{4(39) - 7 = 149^\circ}$$

Practice Problems

5, 7, 13, 14



Standards

CO.10 Prove theorems about and classify triangles. Theorems include:

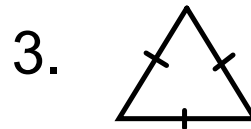
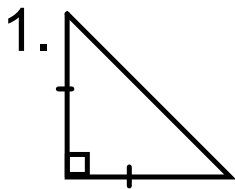
- measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

SRT.4 Prove theorems about triangles. Theorems include:

- a line parallel to one side of a triangle divides the other two proportionally

Review: Put in notes Triangle Theorems

Classify the following triangles by side and angle.



Complete each statement using **always, sometimes or never.**

An Isosceles triangle is _____ an equilateral triangle

An Obtuse triangle is _____ an isosceles triangle.

An interior angle of a triangle and one of its adjacent exterior angles are _____ supplementary

The acute angles of a right triangle are _____ complementary

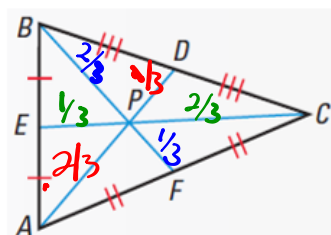
A triangle _____ has a right angle and obtuse angle

Triangle Theorem Notes

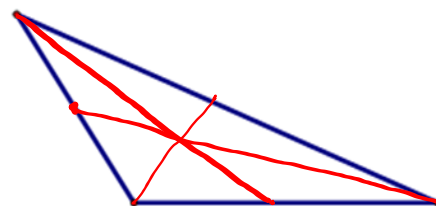
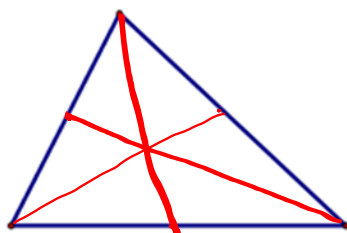
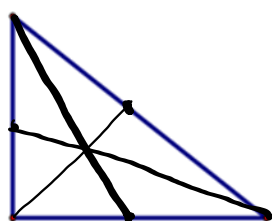
Concurrency of Medians of a Triangle Theorem: The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

If P is the centroid of $\triangle ABC$, then

$$AP = \frac{2}{3}AD, \quad BP = \frac{2}{3}BF, \quad \text{and} \quad CP = \frac{2}{3}CE$$

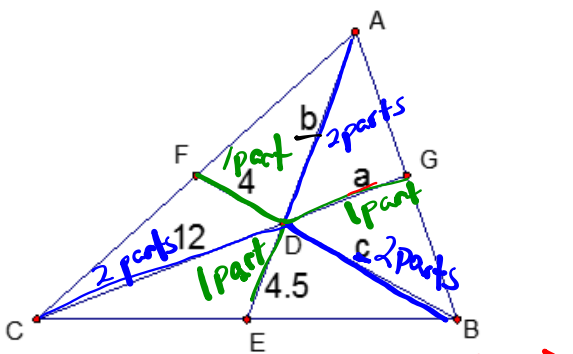


The point of concurrency of the medians of a \triangle is called the centroid.



Triangle Theorem Notes

Using the relationship with centroids, solve for a, b, and c in the triangle.



$$2(4.5) = b$$

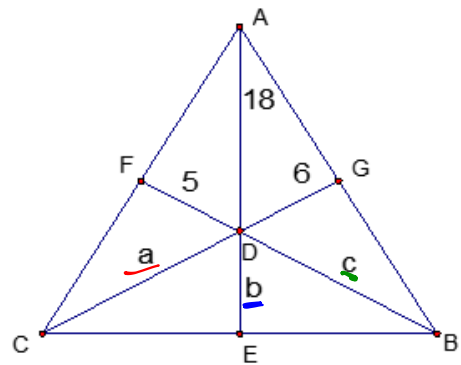
$$9 = b$$

$$2(4) = c$$

$$8 = c$$

$$2(a) = 12$$

$$a = 6$$



$$2(6) = a$$

$$12 = a$$

$$2(b) = 18$$

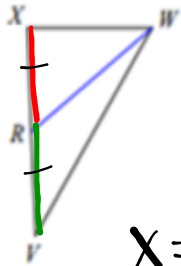
$$b = 9$$

$$2(5) = c$$

$$10 = c$$

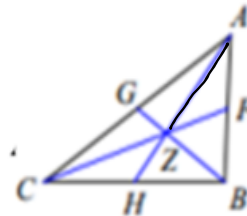
A couple more examples

- 3) Find x if $RV = x$ and $RX = 2x - 4$



$$\begin{aligned}
 x &= 2x - 4 \\
 -x &= -4 \\
 x &= 4
 \end{aligned}$$

- 4) Find x if $AH = -6 + 4x$ and $ZH = x$

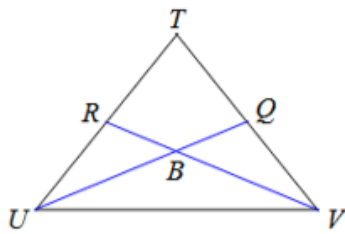


$$\begin{aligned}
 2(x) &= -6 + 4x \\
 -4x & \\
 -2x &= -6 \\
 x &= 3
 \end{aligned}$$

Triangle Theorem Notes

Try some on your own:

- 17) Find x if $VB = 2x$ and $BR = 2x - 3$



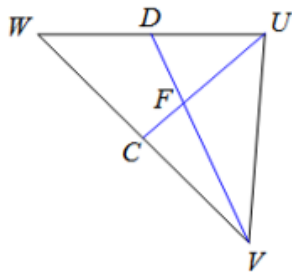
$$2(2x-3) = 2x$$

$$4x-6 = 2x$$

$$-6 = -2x$$

$$3 = x$$

- 19) Find x if $DW = 2x$ and $DU = 3x - 2$



$$2x = 3x - 2$$

$$-x = -2$$

$$x = 2$$

Standards

CO.10 Prove theorems about and classify triangles. Theorems include:

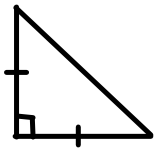
- measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem
 - the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

SRT.4 Prove theorems about triangles. Theorems include:

- a line parallel to one side of a triangle divides the other two proportionally

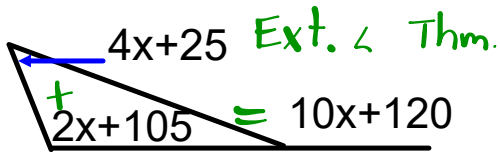
Review

1. Classify the following triangle by angles and sides.

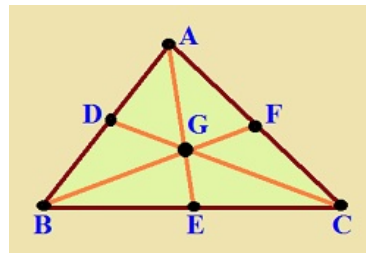


*Isosceles
Right*

2. Determine the value of x . 3. Determine the length of AE .



$$\begin{aligned}
 4x+25 + 2x+105 &= 10x+120 \\
 6x+130 &= 10x+120 \\
 6x &= 10x-10 \\
 -4x &= -10 \\
 x &= 2.5
 \end{aligned}$$



2 parts
 $m\overline{AG} = 10x$
1 part
 $m\overline{GE} = 2x+9$

$$\begin{aligned}
 2(2x+9) &= 10x \\
 4x+18 &= 10x \\
 18 &= 6x \\
 x &= 3
 \end{aligned}$$

Warm Up

The variable expressions represent angle measures of a triangle. Draw a triangle and label the angles. Solve for x and determine value of each angle.

1. $m\angle A = x^\circ$
 $m\angle B = 2x^\circ$
 $m\angle C = 3x^\circ$

$$\begin{aligned}x + 2x + 3x &= 180 \\6x &= 180 \\x &= 30\end{aligned}$$



2. $m\angle D = (3x - 17)^\circ = 3(27) - 17 = 64^\circ$
 $m\angle E = (x + 40)^\circ = 27 + 40 = 67^\circ$
 $m\angle F = (2x - 5)^\circ = 2(27) - 5 = 49^\circ$

$$\begin{aligned}3x - 17 + x + 40 + 2x - 5 &= 180 \\6x + 18 &= 180 \\6x &= 162 \\x &= 27\end{aligned}$$

Standards

CO.10 Prove theorems about and classify triangles. Theorems include:

- measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem
 - the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

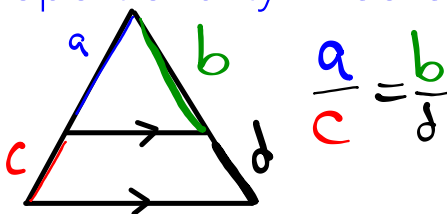
SRT.4 Prove theorems about triangles. Theorems include:

- a line parallel to one side of a triangle divides the other two proportionally

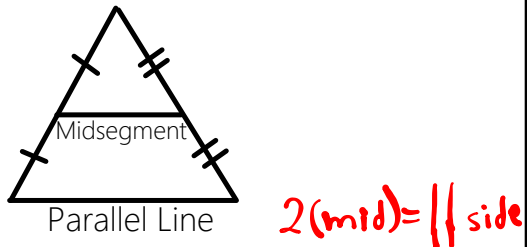
FOLDABLE

Triangle Theorems

Triangle Proportionality Theorem:



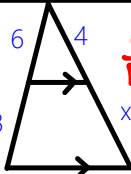
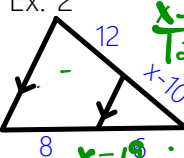
Triangle Midsegment Theorem:

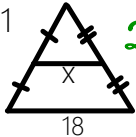
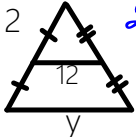


Triangle Theorem Notes

On the inside

Triangle Proportionality Theorem: A line parallel to one side of a triangle divides the other two proportionally, (and its converse).

<p>Ex. 1</p>  <p>$\frac{6}{18} = \frac{4}{x}$ $x = 12$</p>	<p>Ex. 2</p>  <p>$\frac{x-10}{12} = \frac{6}{8}$ $8(x-10) = 72$ $8x - 80 = 72$ $8x = 152$ $x = 19$</p>
--	---

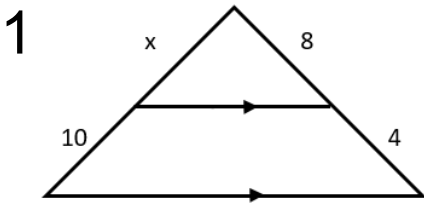
<p>Ex. 1</p>  <p>$2(x) = 18$ $x = 9$</p>	<p>Ex. 2</p>  <p>$2(12) = y$ $24 = y$</p>
---	--

Midsegment of a Triangle:

1. Parallel to one side
2. Midsegment is $\frac{1}{2}$ the length of the parallel side
3. Midsegment contacts the midpoints.

Equation: $\text{midsegment} = \frac{1}{2}(\text{parallel side})$

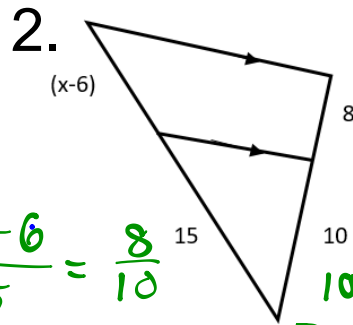
Triangle Theorem Notes



$$\frac{x}{10} = \frac{8}{4}$$

$$4x = 80$$

$$x = 20$$



$$\frac{x-6}{15} = \frac{8}{10}$$

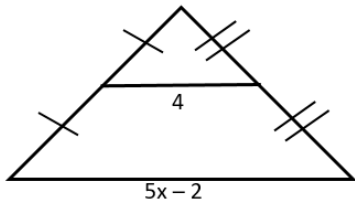
$$10(x-6) = 120$$

$$10x - 60 = 120$$

$$10x = 180$$

$$x = 18$$

3. Solve for x



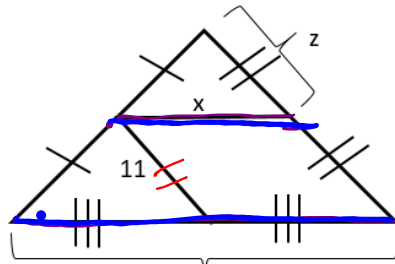
$$2(4) = 5x - 2$$

$$8 = 5x - 2$$

$$10 = 5x$$

$$2 = x$$

4. Solve for the missing variables.



$$2(x) = 24$$

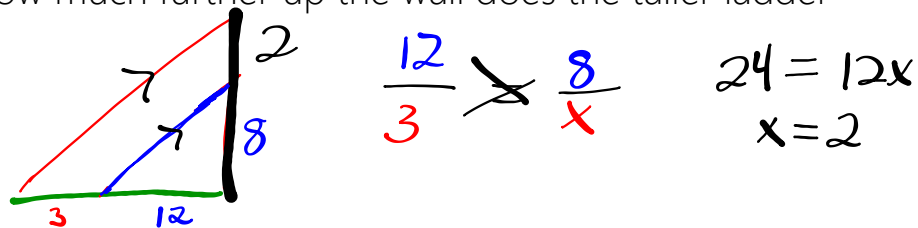
$$x = 12$$

$$z = 11$$

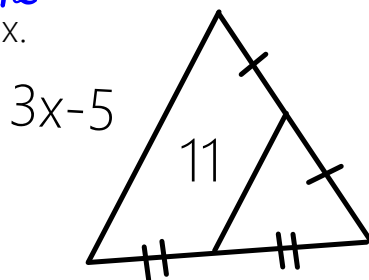
Triangle Theorem Notes

Recap: Similarity and Triangles

1. Two ladders are leaned against a wall such that they are parallel with each other. The shorter ladder's feet are 12' from the wall and reaches 8' up the wall. The taller ladder's feet are 15' from the wall, how much further up the wall does the taller ladder reach?



2. Find the value of x .



Standards

CO.10 Prove theorems about and classify triangles. Theorems include:

- measures of interior angles of a triangle sum to 180 degrees;
- base angles of isosceles triangles are congruent;
- Exterior Angle Theorem
- the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length;
- the medians of a triangle meet at a point.

SRT.4 Prove theorems about triangles. Theorems include:

- a line parallel to one side of a triangle divides the other two proportionally

YAY
GEOMETRY!